Polyhedron, Nobel Prize and Mathematics

Noriaki SUZUKI

Meijo University, JAPAN

The most beautiful temple in Thailand and Japan

Wat Arun (Temple of Dawn)

Kinkakuji (Temple of Golden Pavilion)

The most beautiful mountain in Thailand and Japan

Mt. Chiang Dao

Mt. Fuji

The key ward of my today's talk is beautifulness

The beautiful is very important in Mathematics and Physics

Three topics

[1] Find a beautiful polyhedron with 60 vertexes

[2] Fullerene, LED and Nobel Prize

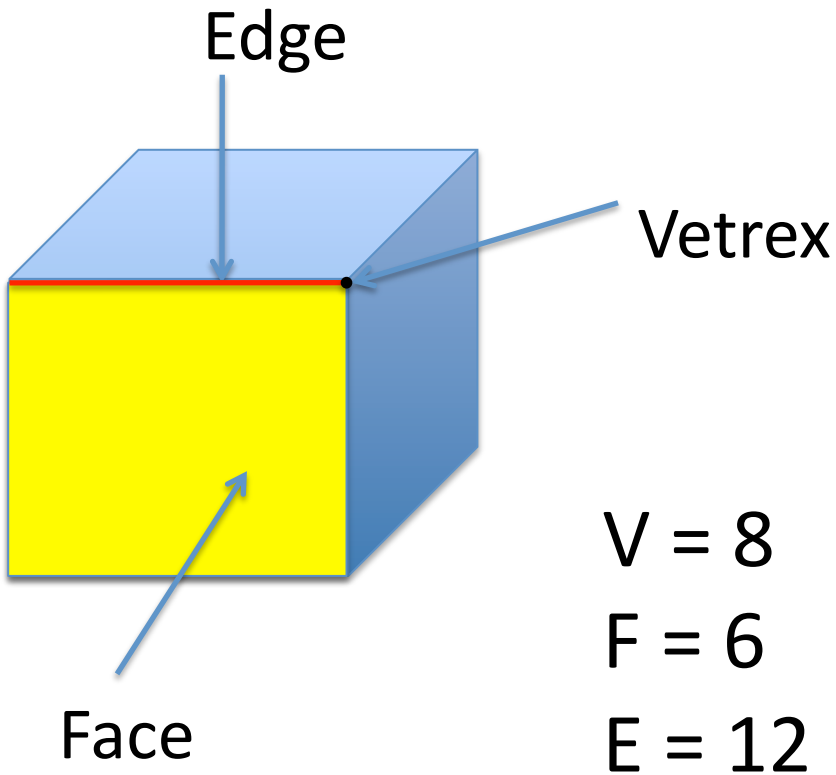
[3] Mathematics helps Physics and vice versa

Find a new dice (regular polyhedrons)

- A traditional dice has six faces showing a different number of dots from 1 to 6. When thrown or rolled, the dice shows dots randomly and each being equally likely.
- For randomly and equally, the shape of dice is it is vertex-transitive, edge-transitive and face-transitive.
- A polygon which has the above property is called a regular polyhedron.

We know 5 regular polyhedrons, and then we have 5 dice

V:vertexes, F:faces and E:edges

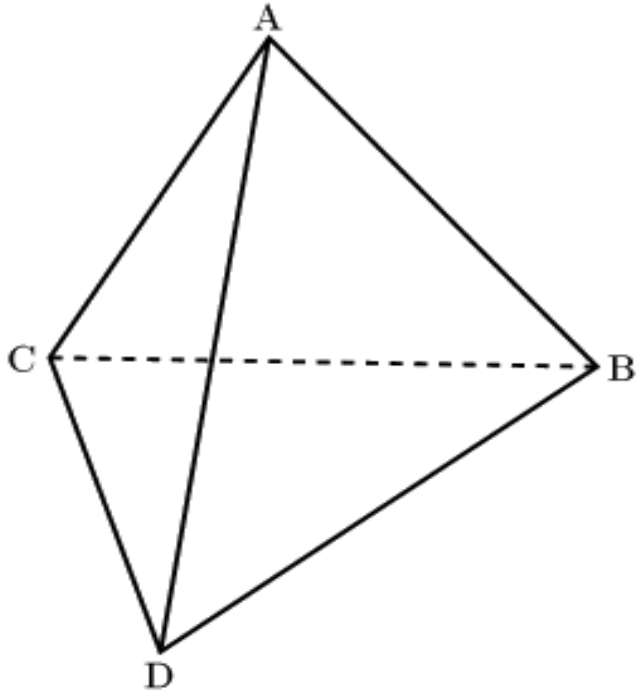


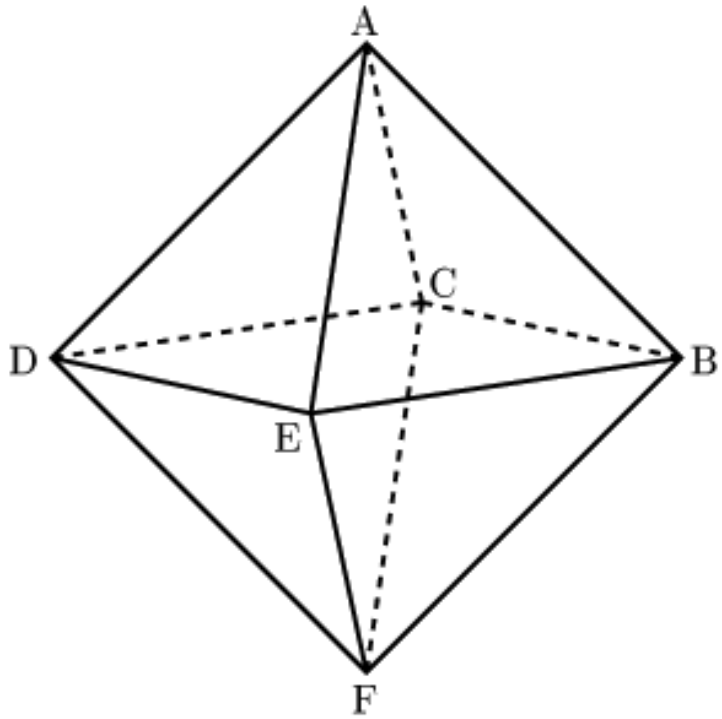
V = 8

F = 6

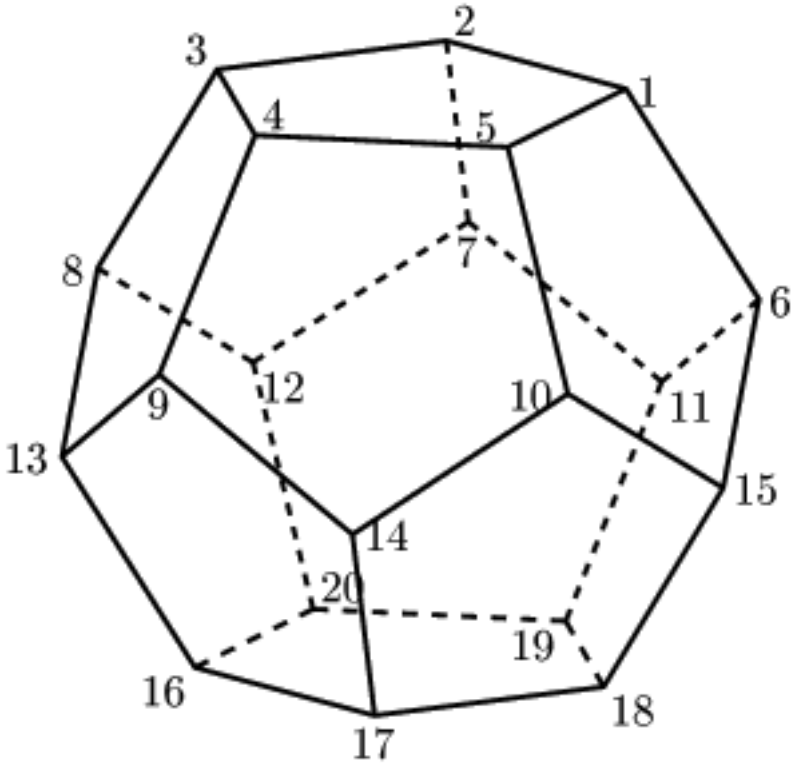
E = 12

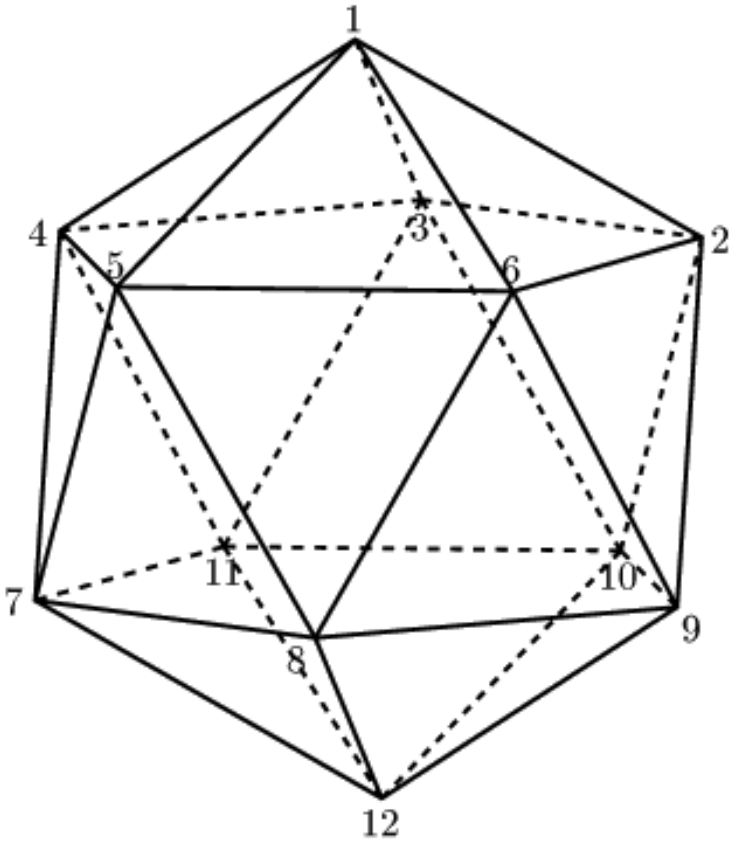
Tetrahedron and Octahedron





Dodecahedron and Icosahedron



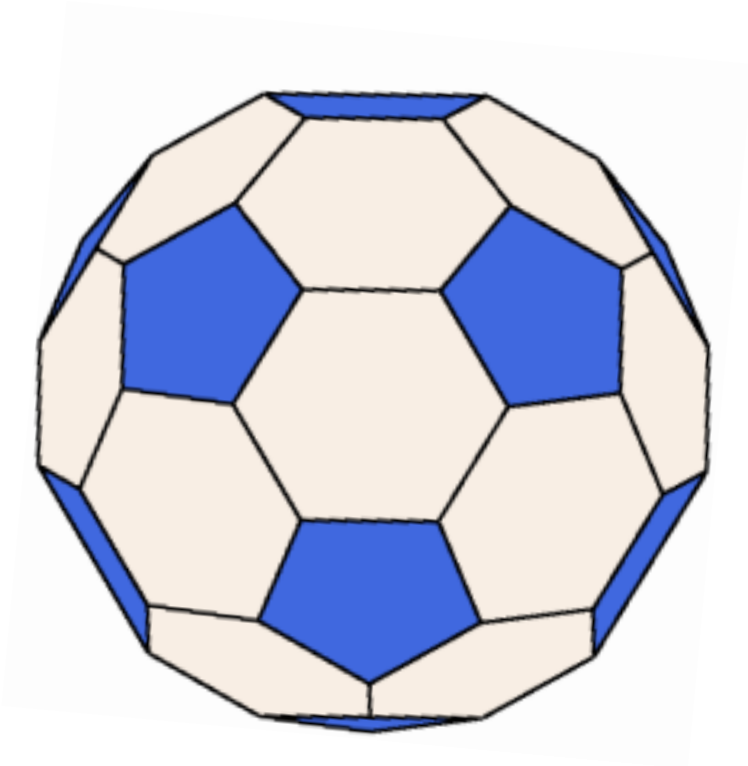


Regular polyhedrons

- Faces of regular polyhedron consist of one kind of regular polygon.
- Tetra = 4 regular triangles
- Cube = 6 regular squares
- Octa =8 regular triangles
- Dodeca = 12 regular pentagons
- Icosa = 20 regular triangles

Pyramid and soccer ball

- Faces of quadrangular pyramid consist of regular 4 triangles and a square, and soccer ball consists of regular 12 pentagons and 20 hexagons.



Beautiful(good shaped) polyhedrons

- Such as a pyramid and a soccer ball, polyhedron whose faces consist two kinds of regular polygons is called quasi-regular.
- Regular polyhedrons are most beautiful ones, and next beautiful ones are quasi-regular.

Number of V, F and E

	V	F	E	V + F - E
cube	8	6	12	2
tetra	4	4	6	2
octa	6	8	12	2
dodeca	20	12	30	2
icosa	12	20	30	2
pyramid	5	5	8	2
soccer ball	60	32	90	2

Descartes-Euler formula

$$V + F - E = 2$$

- All (convex) polyhedrons satisfy the above formula.
- This shows that regular polyhedrons are five. In fact, to determine all polyhedrons is to solve the next problem.
- Also, this formula shows that (non-trivial) quasi-regular polyhedrons are 13.

Equivalent problem

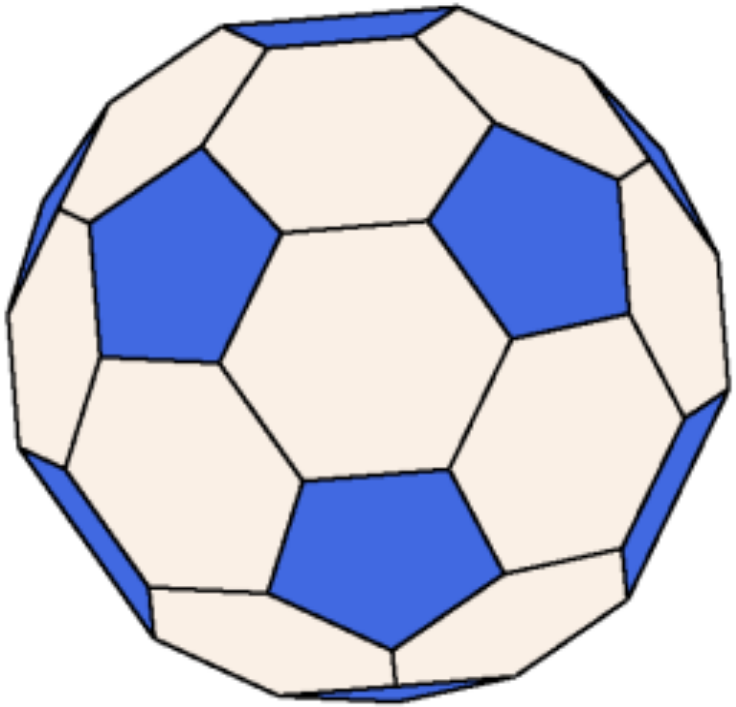
- Let m, n be natural numbers with m, n >2.
  Find all couples (m,n) which satisfy
  $1/m + 1/n > 1/2$
- Solutions: (m,n) = (3,3), (3,4), (3,5), (4,3), (5,3), which correspond to 5 regular polyhedrons.

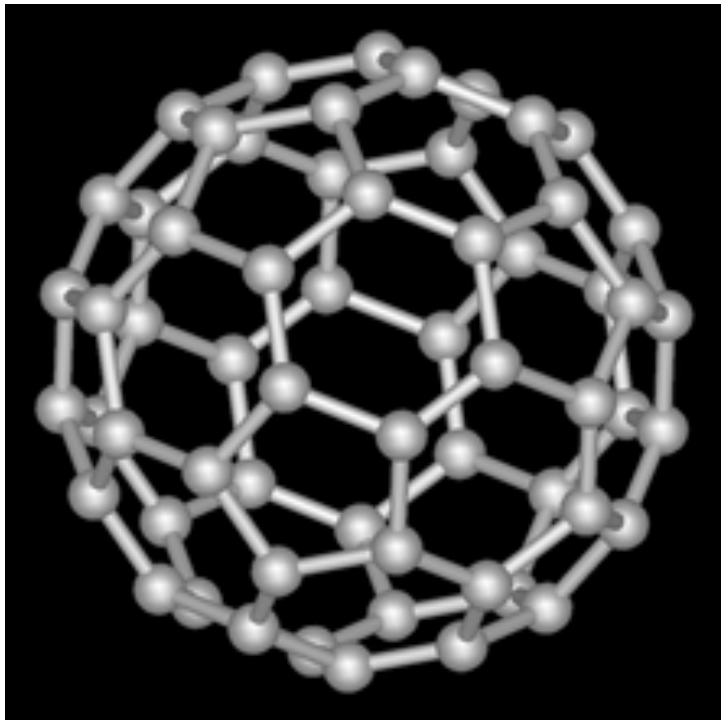
  m: face is m-polygon
  n : each vertex has n edges

New carbon molecule (fullerene)

- In 1985, R. Smalley, R. Curl and H. Kroto discovered a new carbon molecule.
- By the weight, they knew that the molecule made of 60 carbons. Then, what shape is it?
- The universe is well-orderd. All natural objects are beautiful, so new molecule may be good shaped.
- Find a beautiful polyhedron with 60 vertexes.
- They were award the 1996 Nobel Prize.

Soccerball helps Backyball
Mathematics helps Physics



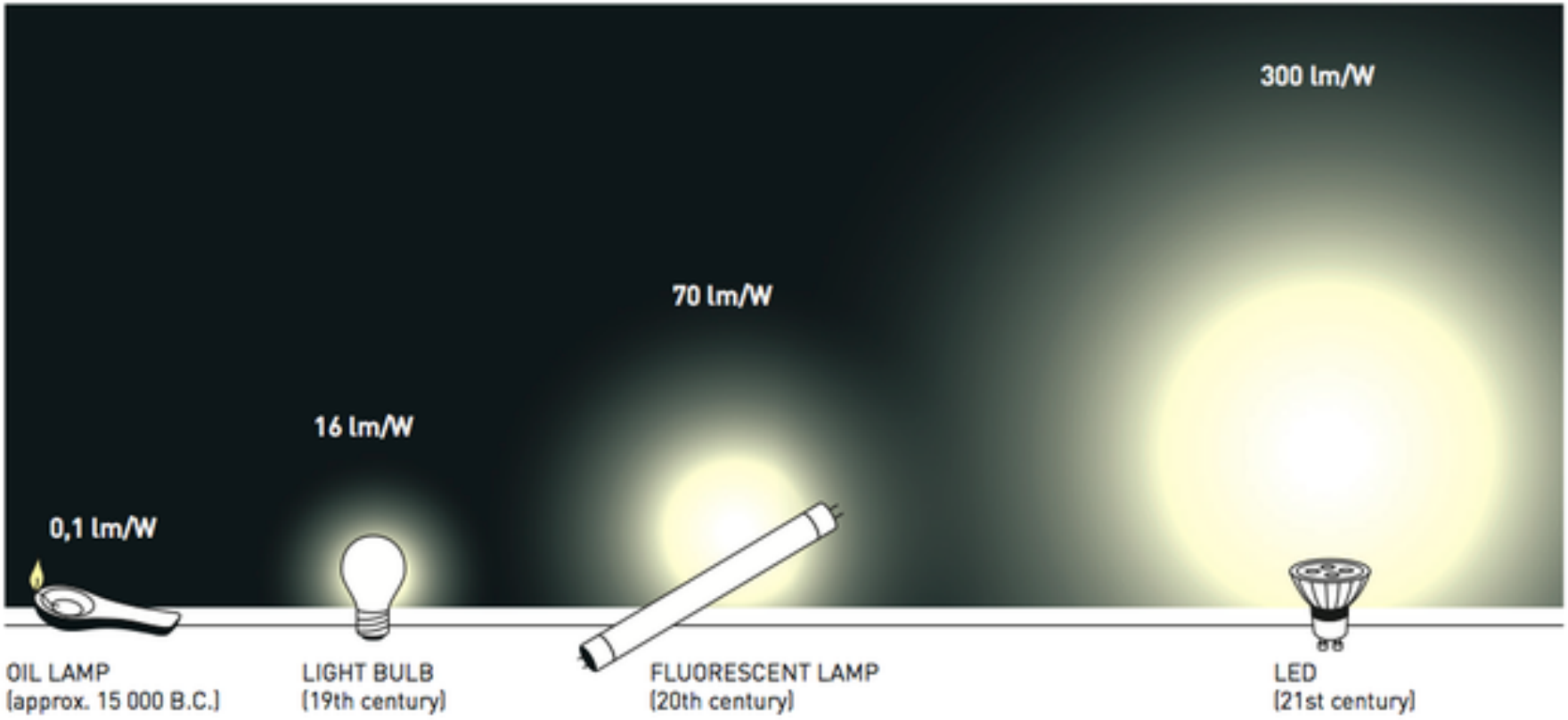


Two famous professor in Meijo Univ.
Isamu Akaski and Sumio Iijima

Professor Isamu Akasaki

- In 1989, Akasaki succeeded to make blue LED.
- The three primaries of the light (red, green and blue make all colors)
- He awards Nobel Prize in 2014

Incandescent light bulbs lit the 20th century, the 21st century will be lit by LED lamps.



Nakamura, Akasaki and Amano

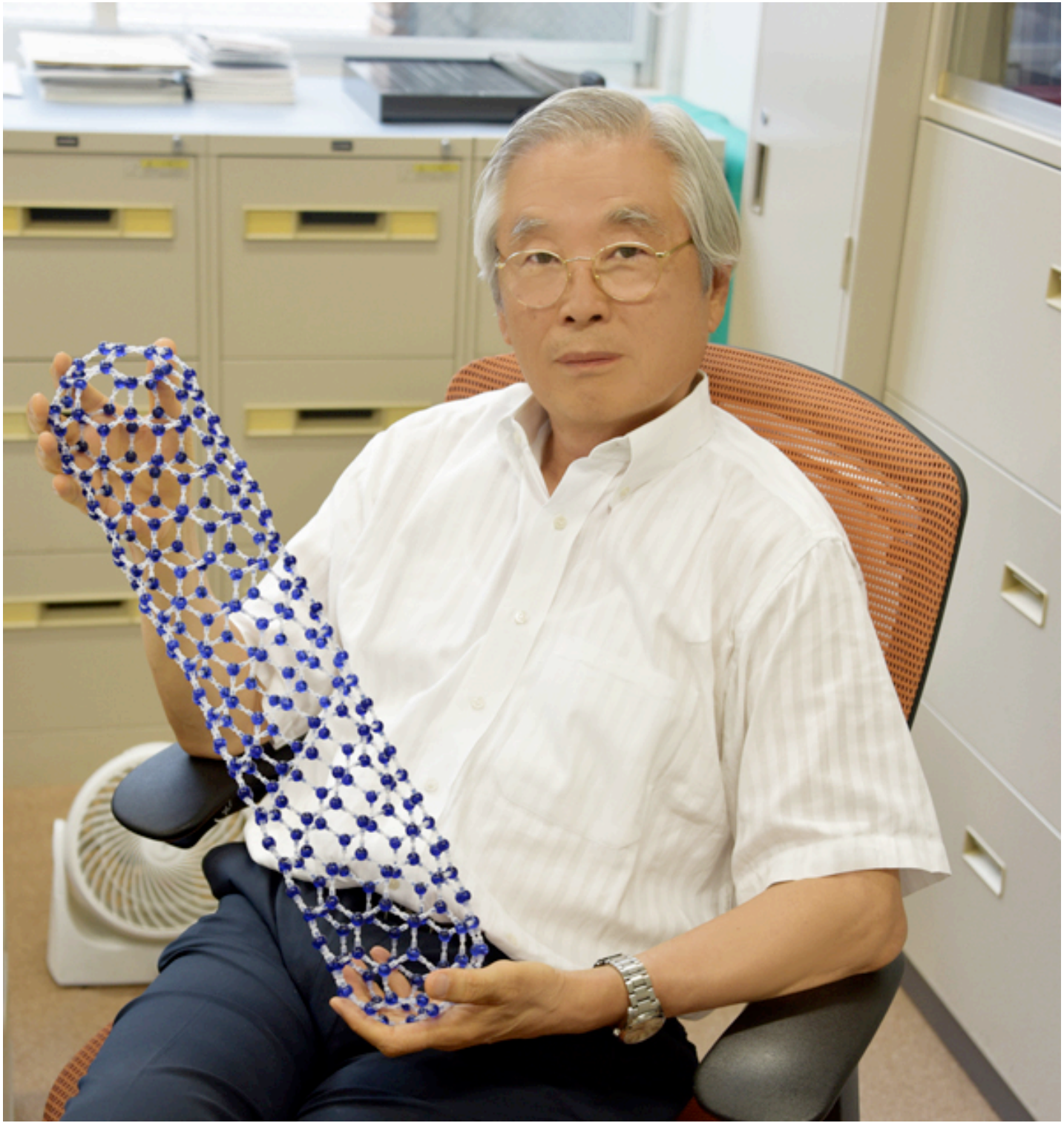
Meijo University

Library of Meijo University

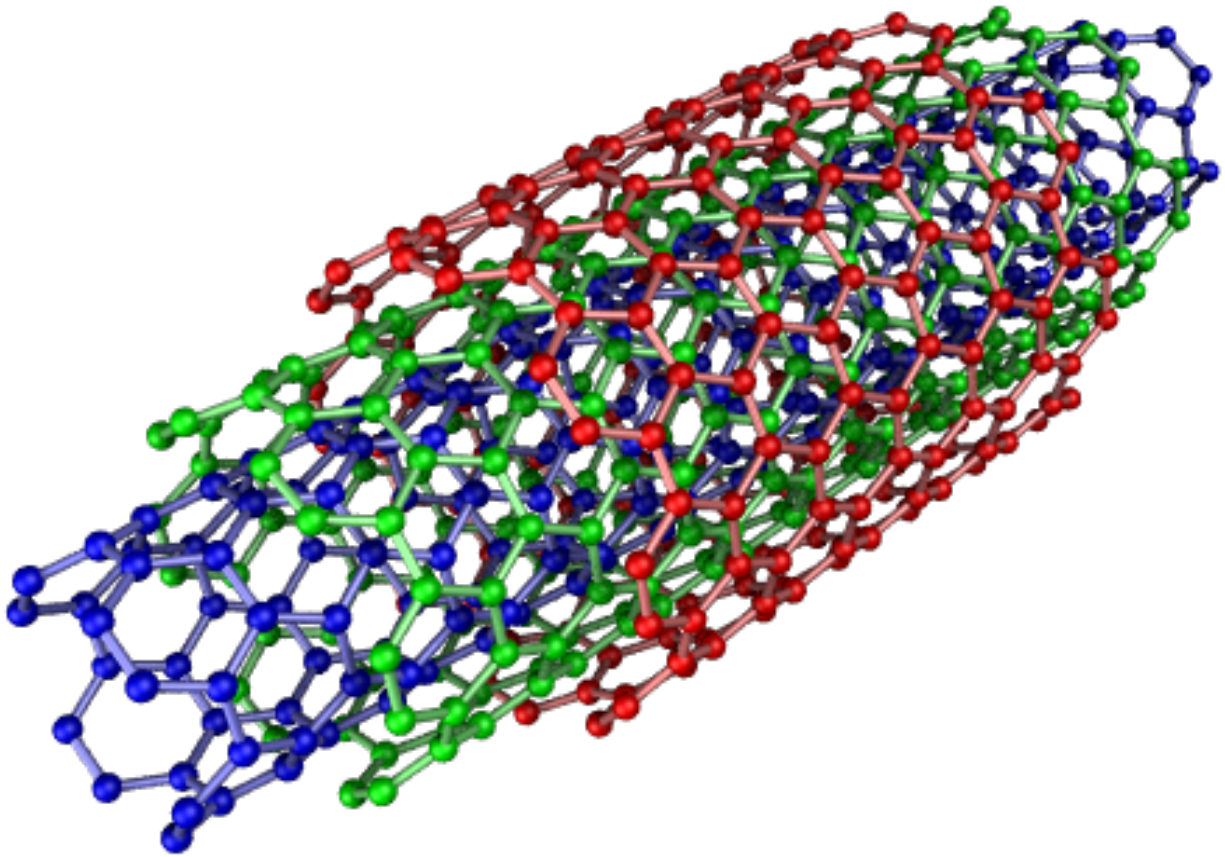
The Entrance of Library
Exhibition of Akasaki's work (Nobel Prize medal)

Professor Sumio Iijima

- In 1991, he discovered carbon nanotube (it is a family of fullerenes) Nano = 10^{-9}
- He wanted to make a large quantity of C_{60}, but he failed. He could not obtain C_{60}, however he could get a new substance. This was the carbon nanotube.
- I think he will maybe get Nobel Prize in near future.



Carbon nanotube



Serendipity

- Iijima discovered Carbon nanotube accidentally, however he had an ability to get good luck.
- In the fields of discovery chance favors only the prepared mind.
- Only a suitable person encounters a great discover.

What is Mathematics?

- Mathematics is thinking.
- Absorb knowledge (to understand big but old theory). Solutions are written in suitable books.
- Create knowledge (to make even small but new theory). Nobody knows a solution.
- There are many familiar problems that nobody knows the solution as yet.

Classify all hexahedrons

- The face of a convex hexahedron is triangle, square and pentagon. We write (a,b,c), if its faces consist a triangles, b squares and c pentagons. For example, cube is (0,6,0) and pentagonal pyramid is (5,0,1). It is not difficult to show that there is no hexahedron with (1,5, 0) (Why? Please consider.)
- Problem: Find all couple (a,b,c) which correspond to some hexahedrons.

Solution

- (6,0,0), (4,2,0), (2,4,0), (0,6,0), (5,0,1), (3,2,1), (2,2,2)
- Further problem: Are there another solution, if we consider non-convex hexahedron.
- I find two another type hexahedrons. I think there are no other solution, but I do not have a proof.

Classification of hexagon

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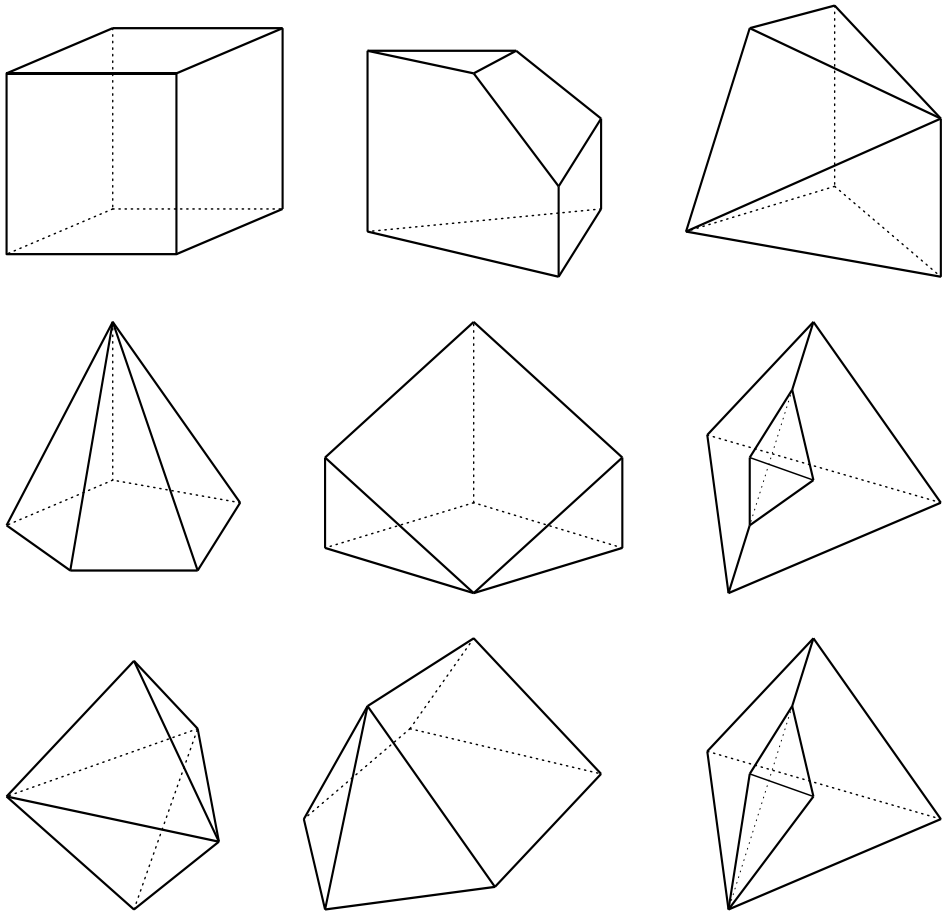
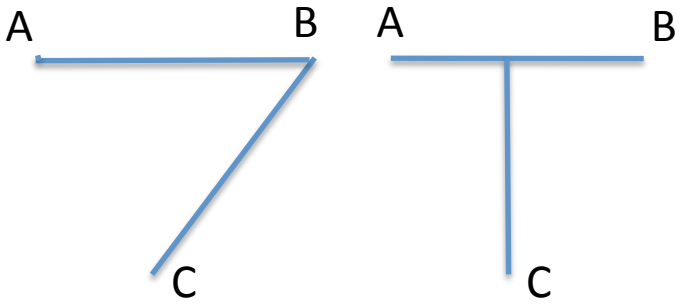


Relation between Mathematics and Physics

- Soccer ball helps C_{60}, Mathematics helps Physics.
- Next example shows that Physics helps Mathematics.

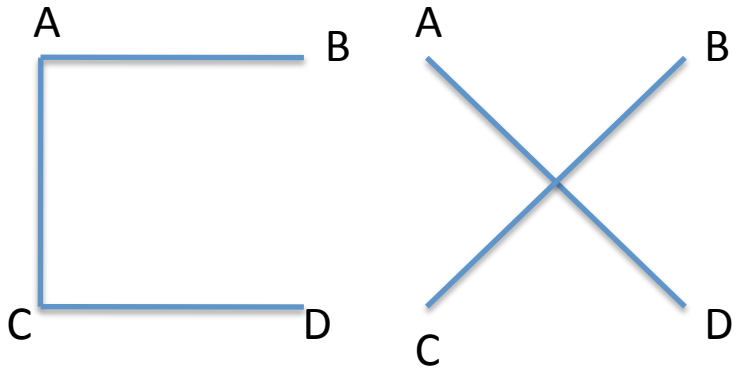
Construct the shortest roads which connects n towns (Fermat-Steiner Problem)

- The case n=3



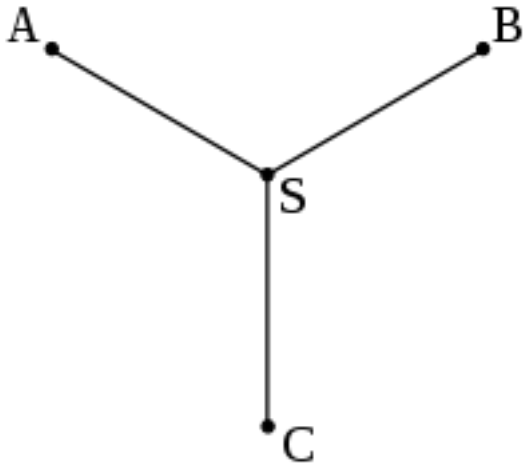
$2, \quad 1+\frac{\sqrt{3}}{2}=1.866\cdots$

The case n= 4

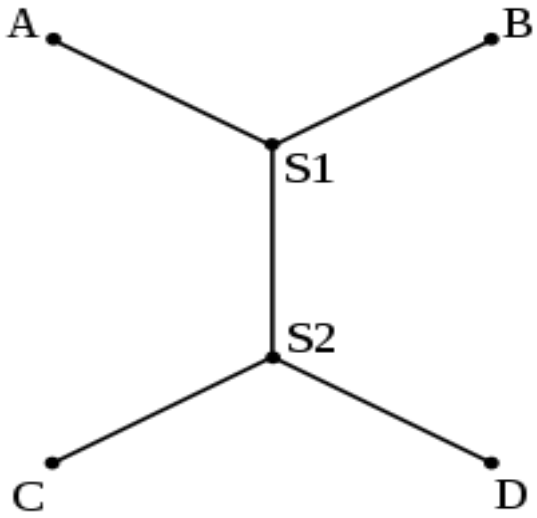


$3, \quad 2\sqrt{2}=2.8282\cdots$

Solutions

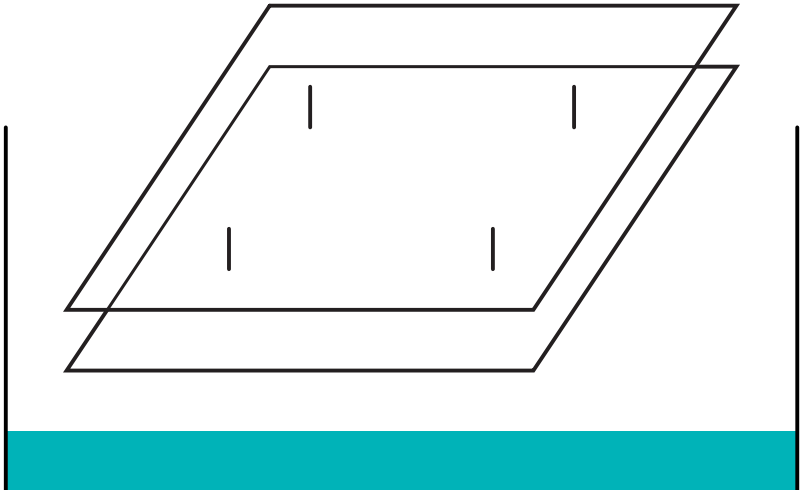


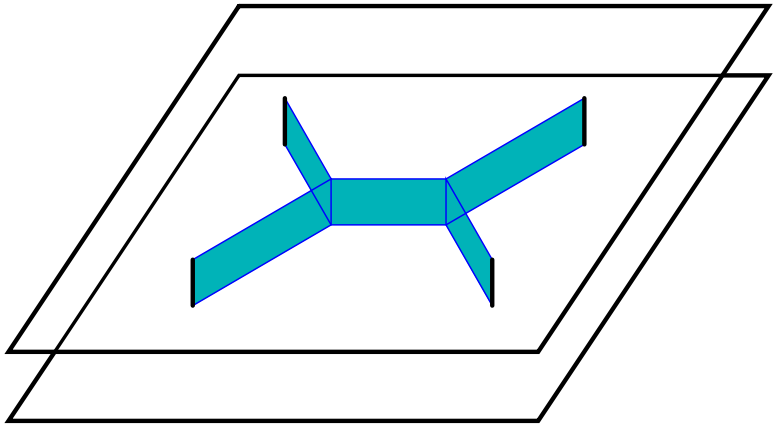
$\sqrt{3} = 1.732\cdots$



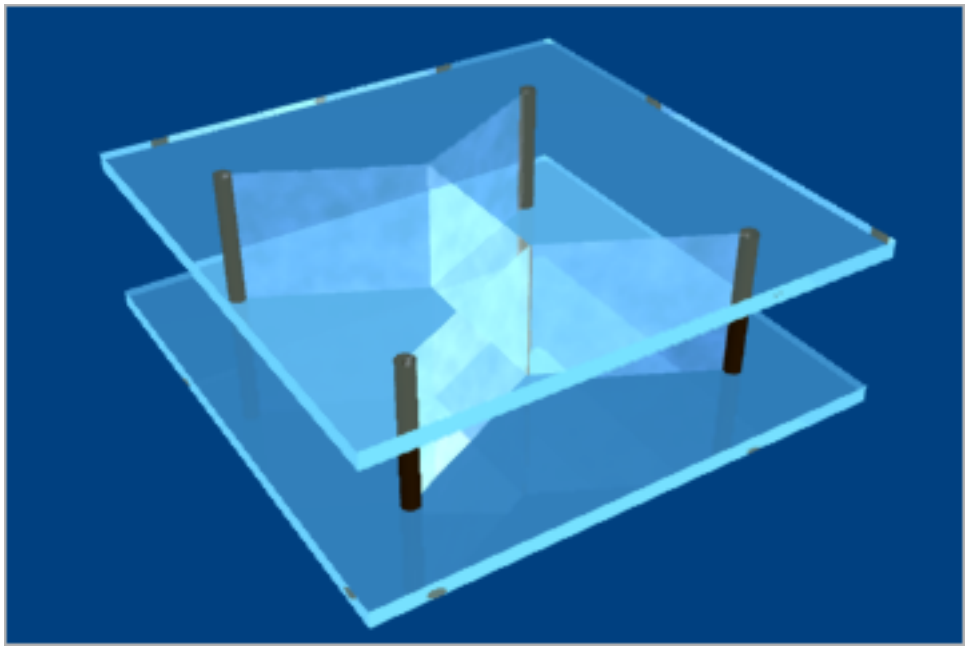
$1 + \sqrt{3} = 2.732\cdots$

Put into soapy water, and then lift up.

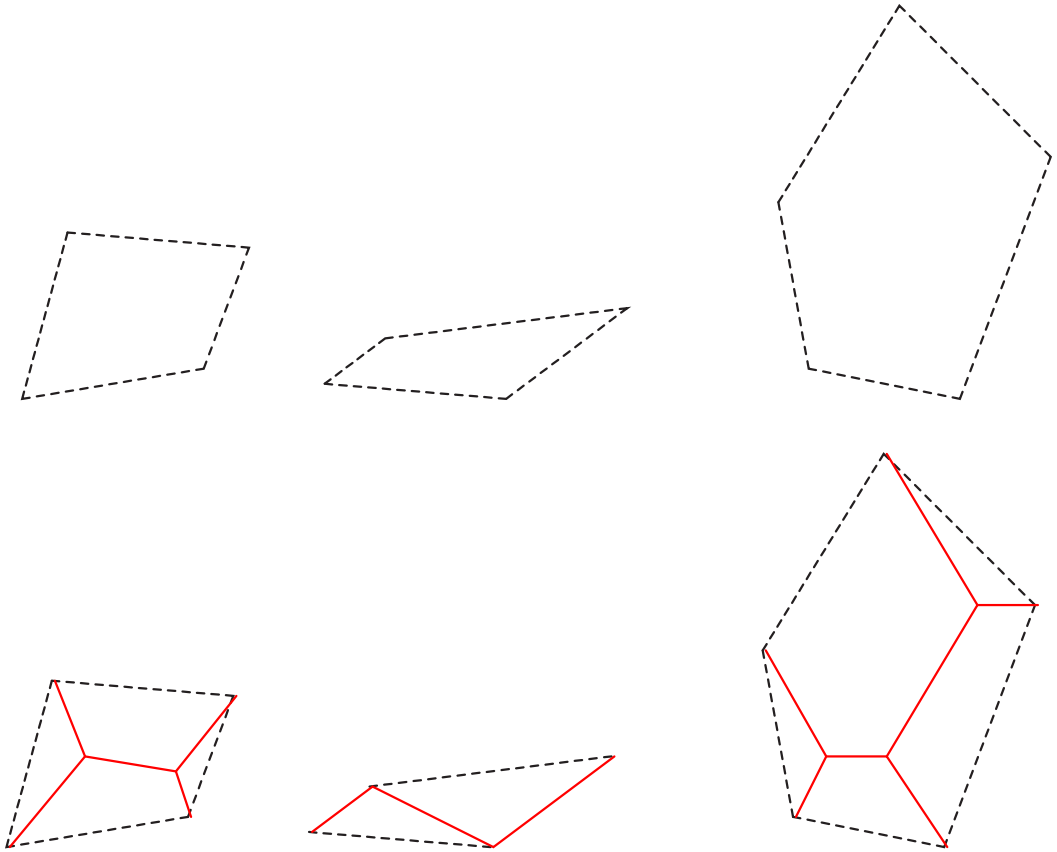




By surface tension, a soap film gives us the shortest road.



Examples of the shortest roads



ขอขอบคุณสำหรับความสนใจของคุณ

(Thank you for your attention)